

AC 2008-1437: A NEW SPIN ON TEACHING 3D KINEMATICS AND GYROSCOPIC MOTION

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A New Spin on Teaching 3D Kinematics and Gyroscopic Motion

Abstract

Students typically struggle with sophomore level dynamics – this difficulty is compounded when the material is extended to three dimensions. Coriolis acceleration can be daunting in planar kinetics – adding additional rotating frames can totally confuse students. Because most courses only offer a rudimentary coverage of three-dimensional motion, textbook problems are typically posed “at an instant in time.” For convenience, the rotating axes are usually chosen to be coincident with the non-rotating axes, and there are seldom rotations about multiple axes. Because of this, students rarely understand the time dependent nature of three-dimensional kinematics and how body-fixed rotating axes systems change over time. To help address this problem, a project was created that requires students to create a model of a motion-based flight simulator. In addition, hands-on mini-labs demonstrating gyroscopic motion were developed to provide a physical feel for three-dimensional kinematics. In the 3D Matlab simulation project, students were first provided a description of how the simulator moves. The team of 3-4 students created a physical model of the simulator with a representation of the different axis systems. This model was used to help the teams develop coordinate transformation matrices between the different axis systems. The angular velocities for each of the different motors (planetary, pitch, and roll) were provided to the student teams and they were asked to determine (a) the angular velocity and acceleration that a pilot in the gondola would experience and (b) the linear accelerations at the pilot’s head (i.e., the vestibular system). After calculating the inertial tensors for the gondola and the planetary arm, students determined the kinetic energy of the system and the moments applied at different bearings in the system as functions of time. We have found that students gain a good appreciation for the time varying nature of 3D kinematics, and understand how to do problems where the rotating and non-rotating axes are not coincident. An animation using the Matlab Virtual Reality Toolbox allowed the students to visualize the kinematic solution and also provided a visual check that the solution is valid. To help students gain a physical appreciation for gyroscopic motion, a set of mini-labs was created. Although most dynamics instructors routinely use a spinning bicycle wheel as a demo, few students get to experience the motion first hand. Similarly, toy gyroscopes can be used to help teach students about precession and demonstrate how gyroscopic navigational devices operate. These hands-on laboratories can be much more powerful than demonstrations and lecturing – the students can actually feel the gyroscopic moments generated. These demonstrations were assessed through two problems on the final examination. The first asked what happens to the motion of a gyroscope when you push gently on the outer gimble. The second involved the action-reaction moments involved with gyroscopic motion (e.g., if you are riding your bike and lean to the left, which way do you have to push on your handlebars). Scores on these different problems along with subjective survey results were used to assess the effectiveness of the mini-labs.

Introduction

Due to time constraints and the difficulty of the material, very few universities include three dimensional kinetics and kinematics in their undergraduate curriculum. We offer a three unit dynamics course (on the quarter system) with a follow-on intermediate dynamics class. This is a four hour course that also includes significant programming with Matlab. A review of 2D kinematics and kinetics is covered during the first four weeks, including an introduction to computer simulation and numerical techniques. The remainder of the quarter is devoted to three-dimensional motion.

In past quarters, the computer simulations did not always match what was being covered in the lectures. Because of the difficulty in performing three dimensional analysis, projects focused on kinematic analysis of linkages and numerical solutions of differential equations. This also exposed students to numerical analysis tools that are not present elsewhere in the curriculum. Recently, we have decided to include a full three-dimensional simulation as a final class project.

During the final week of the course, we cover gyroscopic motion. This is a very difficult concept for most students to grasp, and simple instructor demonstrations don't allow the students to actually feel the gyroscopic moments. To help remedy this situation and hopefully create better conceptual understanding, we created a gyroscope mini-lab. This mini-lab and the final simulation project should provide a better understanding of the time-varying nature of three-dimensional kinetics.

3D Kinematics Project

Calculating three dimensional accelerations can be quite daunting for undergraduate students. Courses that include 3D analysis often only do analyses of "snapshot" dynamics, where the different rotating coordinates line up nicely. In order to promote a more complete understanding of the dynamics, we decided to include a full three-dimensional computer simulation in the class.

The project is an expansion of a project in Tongue and Shepard's Dynamics book¹, and was modeled after the Advanced Spatial Disorientation Demonstrator (ASDD). The ASDD was used for spatial disorientation training in the U.S. Air Force and is shown in Figure 1.

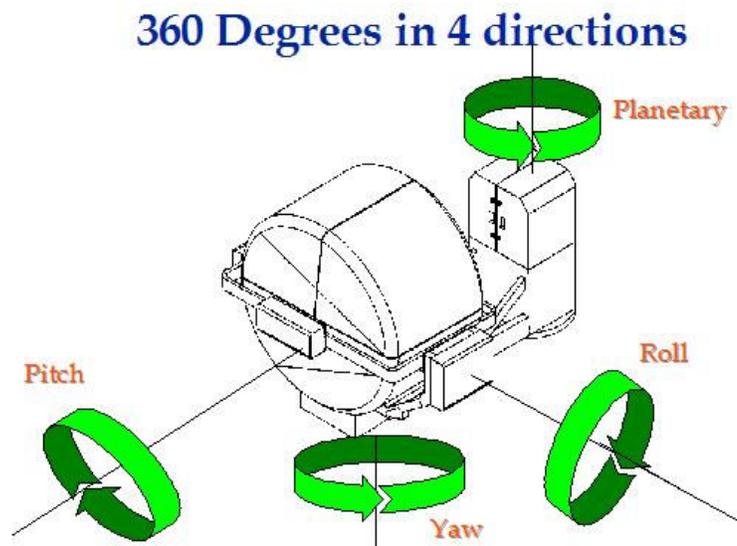


Figure 1. The ASDD.

The device was simplified somewhat, so that there was no yaw motion of the gondola (refer to the full assignment in Appendix A).

To help with visualization, students were required to build a physical model of the device. The model had to show the different coordinate systems used, and how they rotated. One of the more creative models is shown in Figure 2. Other students decided to use more “green” engineering and built models from paper and cardboard. These models were extremely useful for the first part of the ASDD project, where students were asked to compute matrix transformations so that they could calculate vectors in all of the different coordinate systems.

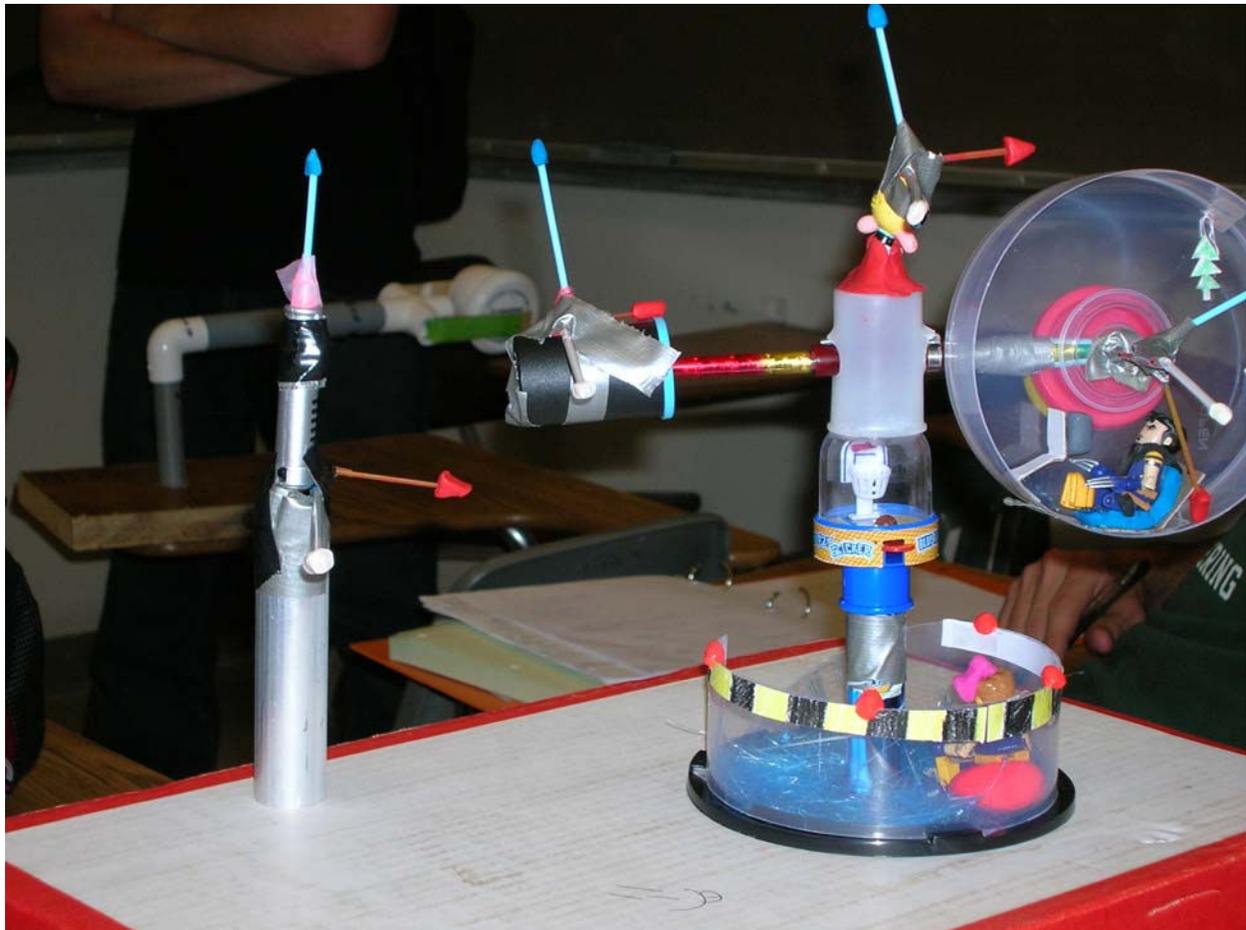


Figure 2. Representative physical model of the ASDD.

Because the ASDD is for spatial disorientation training and research, it is imperative to know the angular velocities and angular accelerations that occur at the pilot’s head. The second part of the assignment required the students to calculate the angular velocity and acceleration of the gondola coordinate system – if the pilot does not move, this coincides to the axis system of the head. This provided motivation for why students might need to calculate variables in multiple coordinate systems (including body-fixed ones). Students were provided the angular velocities applied by each of the drive motors, and were required to use numerical methods to determine the local angular displacements and accelerations.

After verifying that they had the correct angular velocities and accelerations, the students had to calculate the linear acceleration at the head. In addition, the kinetic energy of the entire system and the moments applied to the gondola were calculated. The first two quarters that these were required, mass properties were calculated from dimensions and weights provided by the instructor. Last Fall, the students designed their own device on Solidworks and calculated the mass properties using the software. Representative graphs for the linear and angular accelerations at the head are shown in Figure 3.

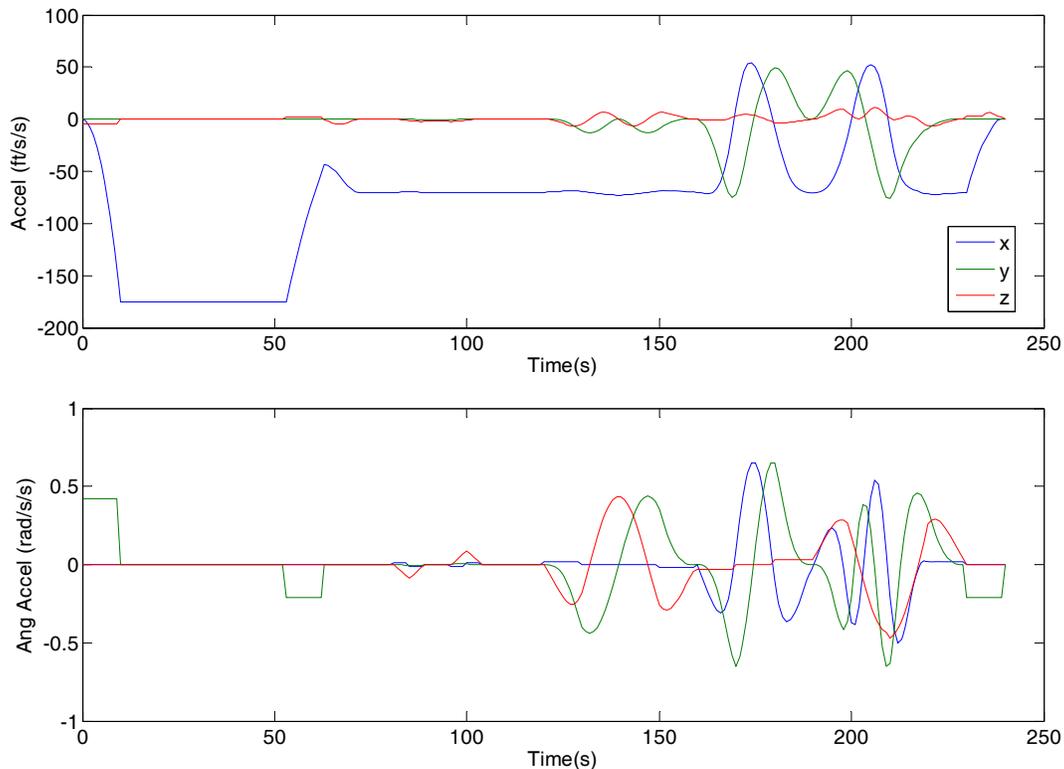


Figure 3. Representative linear (top) and angular (bottom) accelerations.

To verify that their results made sense, we created a simulation using the Matlab Virtual Reality Toolbox. A virtual reality (VR) model was generated with the VRrealm software that included the major components of the ASDD. The VR model was embedded in a Simulink model that connected the motor command signals to the animation. In addition, the animation displayed the three components of acceleration (calculated by the students) in the local gondola coordinates at the pilot's head. This allowed the students to observe the acceleration and do a visual check to see if the results made sense. The model included preset viewpoints that allowed students to view the motion from the fixed coordinates, from the inner arm, outer arm of gondola view. This helped students understand the concept of relative motion and rotating coordinate systems.

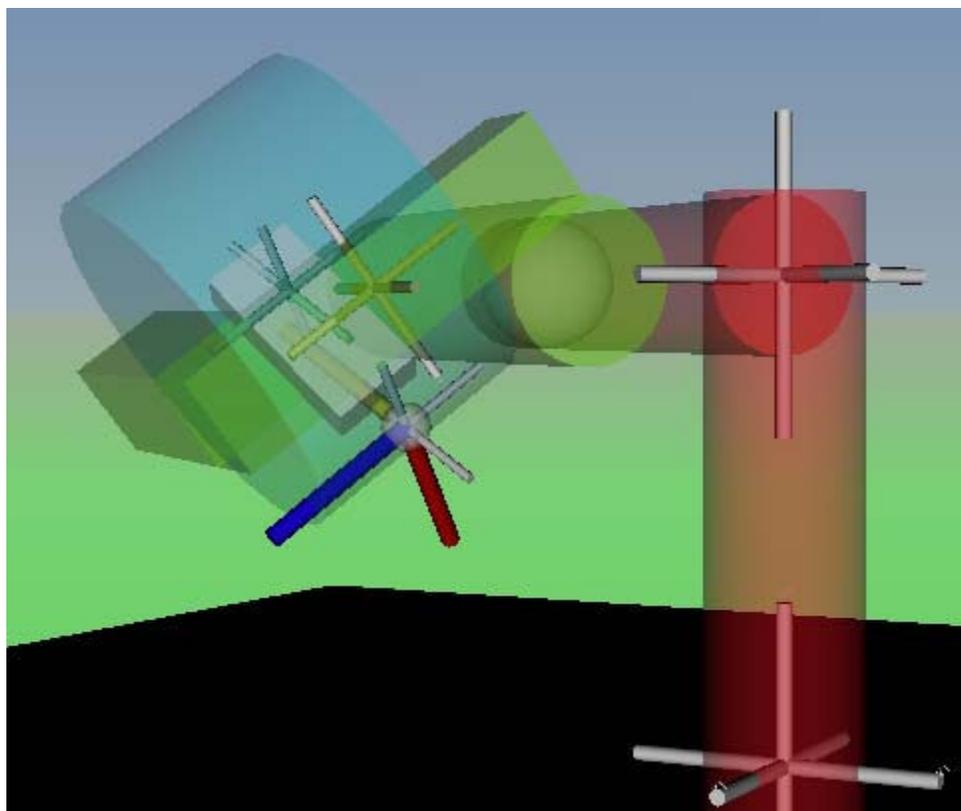


Figure 4. A snapshot of the simulation from the fixed coordinate point of view. Accelerations are represented by the red, blue and yellow bars extending from the pilot's head.

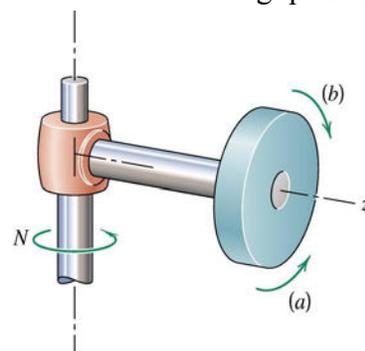
This culminating project applied nearly all of the material that was covered in 3D kinematics and kinetics. Students were required to synthesize the material discussed in class, and to apply it to a real world situation. They were also asked to do some basic research on the vestibular system and predict how the pilot would feel during the simulation.

Gyroscopic Motion

After three-dimensional kinetic were covered, we discussed gyroscopic motion. Based on the reading, students were asked to do a web quiz where they had to answer the following question:

If the disk spins in the (b) direction shown, how does the gyroscopic moment tend to deflect the bar holding the disk (this will be a very small deflection)? All angular velocities are constant.

- (a) The bar deflects upwards
- (b) The bar deflects downwards
- (c) The bar deflects sideways
- (d) The bar doesn't deflect at all



From Meriam and Kraige, 2006².

As expected, students struggled with the concept of the gyroscopic moment and how it affects dynamic systems. It was hoped that this pre-quiz would encourage students to think about the material before the first lecture on the topic. During class, the gyroscopic equations were derived and some example problems worked. A motorized precision gyroscope and a bicycle wheel were also used for demonstration purposes. In the Spring 2007 quarter, 15 toy gyroscopes were borrowed from the Physics Department and the students were told to “play” with them to get a feel for the gyroscopic motion. For the Fall 2007 quarter, we decided to formalize the process with a hands-on lab day. The full assignment is shown in Appendix B, and consisted of four different stations.

Students were divided into groups of three, rotated through the different stations, and answered several questions about each. The first station involved three precision gyroscopes and prompted students to first investigate the simplified gyroscopic equation $\mathbf{M} = I \boldsymbol{\Omega} \times \mathbf{p}$. After spinning the rotor, they pushed different sides of the gimbals to cause precession. As described below, this exercise was directly related to one of the final exam questions. A second exercise was intended to demonstrate how gyroscopic navigational devices work.



Figure 5. Precision gyroscope station.

The second lab station consisted of several toy gyroscopes. Students were instructed to hold the gyroscope and slowly move it so they could feel the gyroscopic moment. The second exercise was to investigate precession and nutation by placing the spinning gyroscope on a small stand.



Figure 6. Toy gyroscope station and bicycle wheel station.

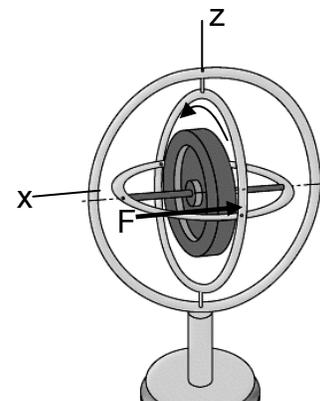
Six different bicycle wheels and two rotating platforms were used for the third station. This is one of the best ways to truly feel the gyroscopic moment – each person held the spinning wheel while they tried to rotate about different axes. The students were also asked to essentially replicate the online quiz mentioned above. With the wheel spinning in the (b) direction, they spun to their left. They can then experience for themselves that their arms deflect downward due to the motion (i.e., they have to exert an upward moment for the rotor to spin and precess in the prescribed motion).

The final station was a tour of the satellite simulator system in the Cal Poly Space Technologies and Applied Research Laboratory. The research project uses four different inertia wheels driven by motors to control the attitude of a satellite structure supported on a spherical air bearing which simulates a gravity free environment. The students got a first hand view of how gyroscopic effect is used in satellite systems.

Assessment

Two different final exam problems were used to assess changes between the Spring (no gyroscopic mini-lab) and Fall (gyroscope mini-lab) quarters. The results of both groups were assessed to compare the effect of the hands on experience. It was anticipated that the experience would help give students a greater intuitive understanding of the relationship between spin, precession and the applied moment. Problem 11 is shown below:

The gyroscope at the right has a spin direction as shown, with the angular velocity in the positive x direction. If you push gently on the outer gimbal in the negative x direction (shown as force F), what will happen (be specific)?



The student answers were graded as follows:

- Score of 5 was given if students recognized that the result was a precession of the disk, could apply the correct equation, and compute the axis and the correct direction of the precession.
- Score of 4 was given if students did everything, including finding the correct axis, but gave the wrong direction of the precession.
- Score of 3 was given if the students recognized that the result was a precession, wrote the governing equation but then applied it incorrectly.
- Score of 2 was given if the students either recognized precession, or wrote the governing equation but not both.
- Score of 1 was given if student made a basic observation that the force caused a moment, or made some other basic observation.
- Score of 0 was given when the student gave no response or the response showed no understanding of the system.

The results of the assessment of Question 11 are shown in Figure 7. The overall average increased a from 3.23 in the Spring (no lab) 3.45 in the Fall (lab). A Mann-Whitney rank sum test showed that there was no statistically significant different between the two groups ($p=$

0.224), but we were encouraged by the large increase in the number of students that received full credit after the hands on experience. Unfortunately there was also an increase in the number of Fall students who scored a 2 on the problem. These results suggest that the exercise may have helped some students gain a richer understanding of gyroscopic systems, but many are still struggling with the concept.

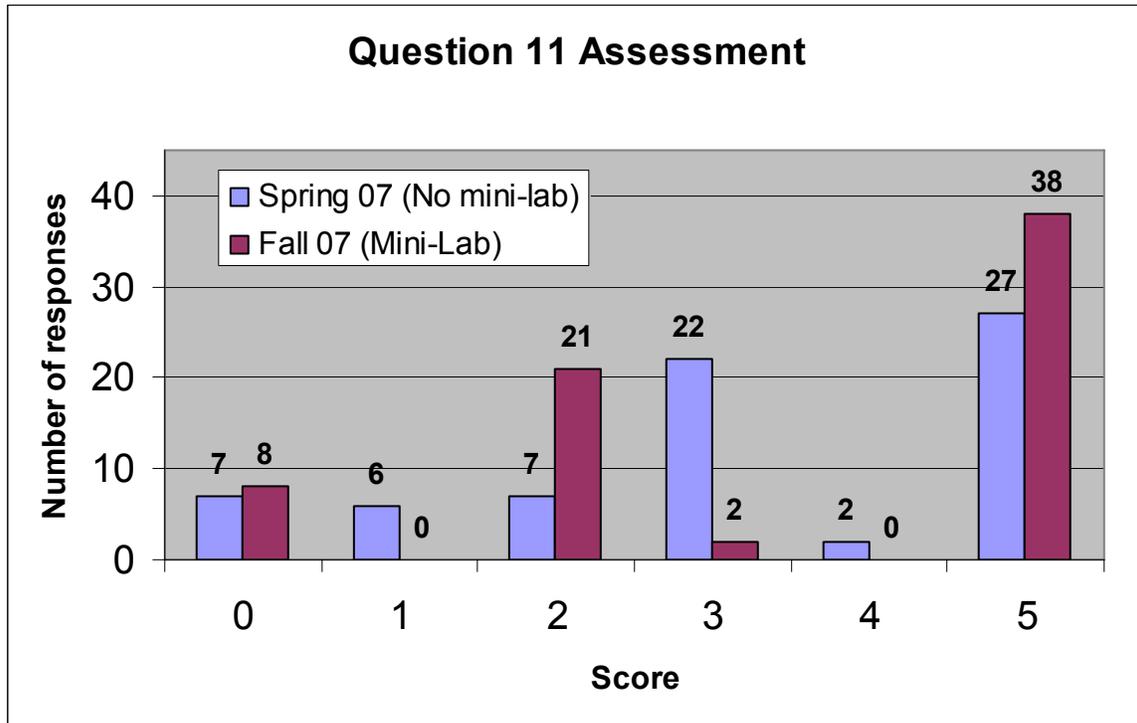


Figure 7. Scores for question 11.

The second problem deals with the gyroscopic moment caused by a jet turbine spinning, as shown in Figure 8. For our assessment, they were given the following scores:

- Score of 2 was given if the axis and compensation decisions were both correct
- Score of 1 was given if students found the correct axis, but drew the wrong conclusion about how the pilot should compensate
- Score of 0 was given if students didn't find the correct axis of the moment (k)

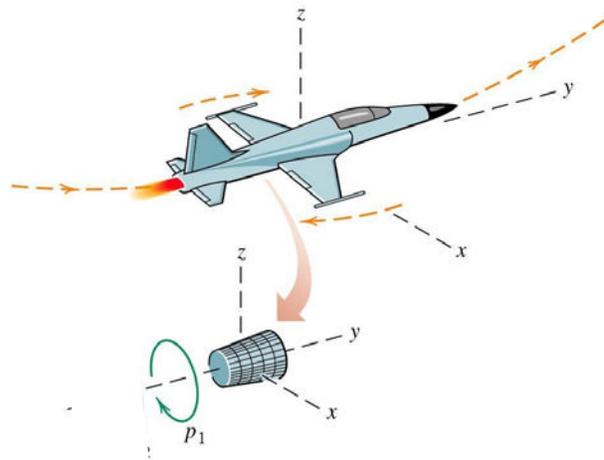


Figure 8. Rotating turbine in jet. From Meriam and Kraige, 2006².

The results are shown in Figure 9. There appears to be a slight improvement in understanding, as the average score increased from 2.17 to 2.26. A Mann-Whitney test again showed no significant difference ($p=0.207$), which is not surprising given the relatively small sample size.

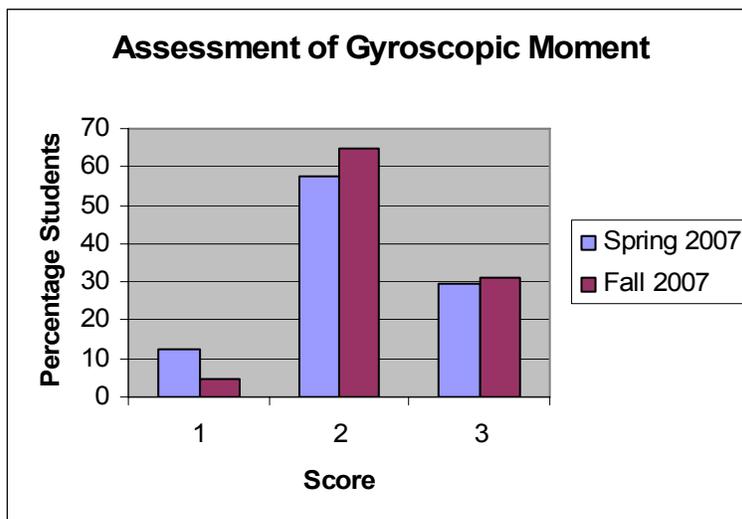


Figure 9. Scores on gyroscopic effect of rotating jet engine.

Survey Results

An end of course survey was given to students during the Fall 2007 quarter. Using a five point Likert agreement scale, students rated four different aspects of the course: Homework, a hands-on catapult project³, the ASDD project, and the Gyroscope mini-lab.

The [HW/Catapult/ASDD/Gyroscope] helped me learn the course material
The [HW/Catapult/ASDD/Gyroscope] was interesting and motivating.

The catapult project involves taking mass properties measurements, performing a Matlab simulation to calculate the angular velocity as a function of the arm angle, and predicting the range of the projectile (a raw egg). It is one of the more popular projects, and is included here for comparison.

The percentage of students supplying each response is provided in Figure 10. It can be seen that over 90% of the respondents thought that the ASDD project helped them learn the course material (strongly agree or agree). Surprisingly, this rated quite a bit higher than the homework assignments, which typically score high for helping students learn the material. As expected, the catapult project had the highest scores for interest and motivation, although the ASDD project and the gyroscope mini-lab also appeared to have motivated the students.

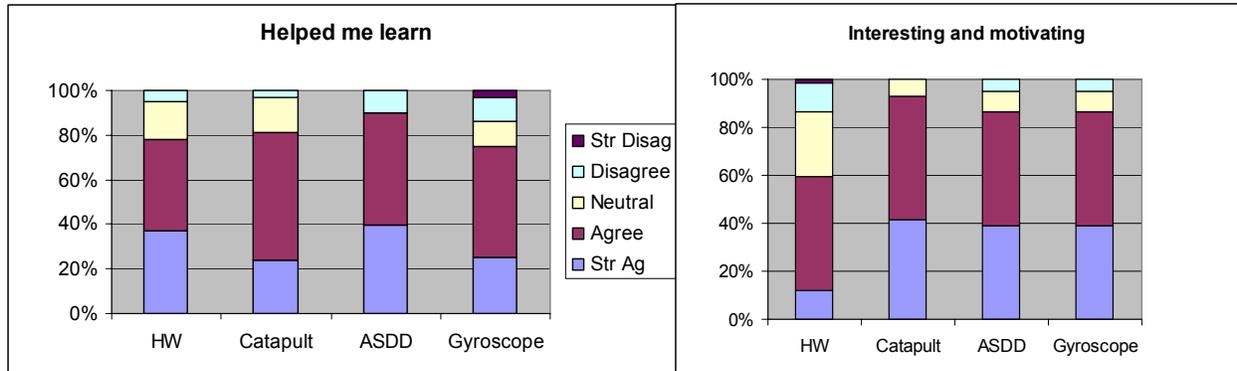


Figure 10. Subjective survey responses.

An additional question addressed the ASDD project:

The ASDD project helped me visualize and understand three dimensional dynamics.

The results are provided below.

Strongly Agree	Agree	Neither Agree nor Disagree	Disagree	Strongly Disagree
21	30	4	4	0

Finally, an open ended question inquired about the gyroscopic mini-lab:

The gyroscopic mini lab could be improved by...

There were three primary themes to the answers. Eighteen respondents thought that the lab was too rushed and that they did not have enough time to do the lab and fill out the questions. Seven students thought that there should be more lecture on gyroscopes before the mini-lab, and thirteen said that the lab was fine the way it was. A few representative comments follow.

Do some questions that relate between the experiences there, and example problems from the book so that you can see what you're experiencing has a real impact. The way it was, it had more the character of some fun play time (which isn't a bad thing).

I liked it. Got to see first-hand how gyros worked. MUCH more useful than the related online quiz.

Maybe a summary at the end to clarify what the students just witnessed/experienced.

Possibly having more people (upper division ME students?) to explain how gyroscopic motion is working at each station.

Conclusions

Overall, the gyroscope mini-lab and the 3D kinematics project appear to improve student learning and motivation. Although students struggle with the Matlab simulation, they recognize that modeling the time-varying accelerations of the device provides them with a better understanding of three-dimensional motion. It is also important to note that most student teams were able to accurately calculate the linear and angular accelerations at the head, as well as the kinetic energy of the system and the applied moments at the gondola.

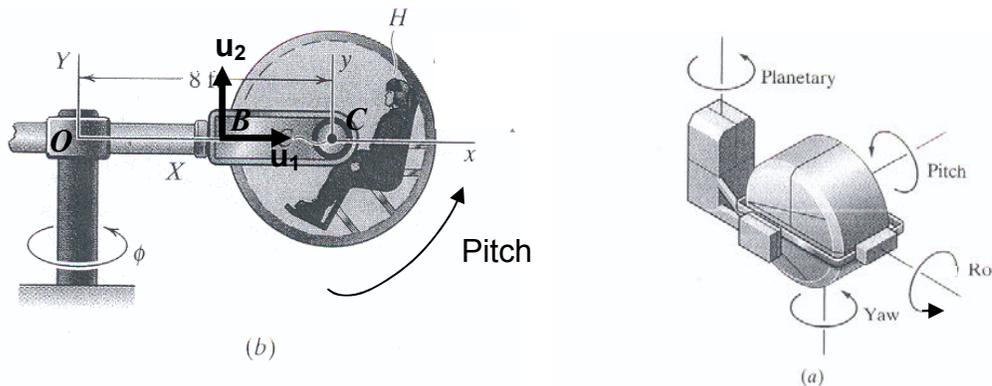
Most students also appreciated the gyroscope mini-lab. Although overall performance on exam problems did not improve greatly between Spring and Fall quarters, there was a substantial increase in those who fully understood the question based on a precision gyroscope. The students also made several suggestions to improve the lab, including having a graduate student present to help explain the labs, linking the lab to additional homework problems, and providing a summary to clarify what was done in the lab. By implementing some of these suggestions, we hope to see even greater improvements in future quarters.

Bibliography

1. Tongue, B.H. and Sheppard, S.D. (2005) Dynamics: Analysis and Design of Systems in Motion, John Wiley & Son.
2. Meriam, J.L. and Kraige, L.G. (2006) Engineering Mechanics, Volume 2, Dynamics, 6th Edition, John Wiley & Son r.
3. Self, B.P. and Redfield, R. (2001) New Approaches in Teaching Undergraduate Dynamics. Proceedings, American Society for Engineering Education Annual Conference and Exposition.

Appendix A – ASDD Parts 2 and 3

Examine the figure below. Set up a coordinate system $N_1-N_2-N_3$ that is fixed to the earth. Set an XYZ system that rotates with the arm through the planetary angle ϕ . At the attachment to the vertical shaft (point O), the arm can actually rotate about the u_1 axis (this would roll the pilot). Attach an intermediate axis system $u_1-u_2-u_3$ on the arm that rolls with the arm through an angle θ . Finally, attach an xyz system that rotates with the cab (e.g., it pitches along with the pilot) through an angle β . This setup is exactly like what you did for Part I. As you start your Matlab program, make sure that you can adjust your transformation matrices in case you did Part I incorrectly.



You need to determine the angular velocity and angular acceleration of the cab in terms of $\dot{\phi}, \ddot{\phi}, \dot{\beta}, \ddot{\beta}, \dot{\theta}, \ddot{\theta}$. For the hand calculations that you will turn in, you can just write it in terms of these angular velocities/accelerations and the local coordinate unit vectors (i.e., it is okay if you leave the unit vectors as u_2 or an \mathbf{i} or a \mathbf{j}).

On Blackboard under Assignments is an Excel sheet: *asdd-f07.xls*. It has angular velocity profiles that are given by each of the motors – planetary, pitch, and roll. The first column is time. These are provided in RPMs in their local systems (because the motors that drive the motion are in these local systems). They are all positive with respect to the coordinate systems set up during Part I. You will have to use the transformation matrices from Part I to get ω and α in the ijk system.

For Part III, you will also be given the length that the arm OC extends (variable L). The base length is 8 feet – you will add the length L to get the total length from O to C . You won't need this for Part II, but will need it for Part III where you will have to calculate the linear acceleration of the head, as well as the kinetic energy of the system and the moments at specified locations.

ASDD Part III

As discussed in Parts I and II, the human vestibular apparatus is an extremely complex system. In order to develop meaningful motion profiles in devices such as the ASDD, it is necessary to know the angular and linear accelerations at the head. The head is located at $2\mathbf{i} + 3\mathbf{j}$ (feet) when measured with respect to point C – you will use the different angular velocity profiles from *ASDD-F01.xls* in this project.

The rate of extension of the arm will be provided on Blackboard in the file “*ASDDarm.xls*” The first column is time in seconds, the second column is the extension rate in feet per second. Using your previous calculations for the angular velocity and angular accelerations, plot the angular and linear acceleration at the head in the *ijk* system as a function of time (break into coordinates again). These values are important when trying to determine the perceived accelerations as a function of gondola accelerations.

Design the ASDD on *Solidworks*. It must be able to do the different motions listed (roll, pitch, and extension), and the maximum length from point O to point B will be 8 feet. The distance between point B and point C is always 5 feet. You can model the occupant as a 150 lbs point mass at point C. Use *Solidworks* to calculate your inertia matrix – it would be smart to use planes of symmetry in your design to simplify this matrix. Don’t worry about adding in the weights of the motors, or other equipment, just make the cylindrical shell for the gondola and create a mechanism for the arm that will allow it to extend. Make sure you discuss your design and list your inertial matrices for each part in the ASDD. You should also discuss what structural analysis you would need to do on your design (if we had the time).

Your report should include plots (vs time) for the following:

- (a) the linear and angular accelerations at the head
- (b) the kinetic energy of the gondola alone and of the entire ASDD
- (c) the moment applied to the gondola at point C

You will also be required to calculate the location of points B, C and H (the head) in the non-rotating reference frame. This will serve as input to a simulation that Dr Birdsong (expert consultant to the project) is creating. He may require additional inputs as the project progresses – more detail at a later date.

Your final turn-in is due Wednesday, Nov 28th and should be in the familiar Report format. You should include the hand calculations that you did in Part II (corrected if necessary). Do some background research on the vestibular system to find minimum thresholds when you detect angular and linear motion (about a page of background information is sufficient – make sure you cite your references). You should also make sure this profile doesn’t go beyond maximum thresholds that can safely be endured by your normal Cal Poly M.E. student. Based on your research, estimate how you think someone might feel throughout the profile (at a minimum you should discuss the Coriolis illusion). Mention practical applications where knowledge of vestibular function (and developing vestibular models) might be useful. Finally, suggest a different type of design that might be used for a ground-based flight simulator.

Appendix B - Gyroscope Mini-Lab (you will turn this in at the end)

TEAM  _____ 3PM 4PM

As you remember from lecture, $\mathbf{M} = I \boldsymbol{\Omega} \times \mathbf{p}$ -- I and \mathbf{p} are about spin axis, $\boldsymbol{\Omega}$ denotes precession

Gyroscope (Gyro on the schedule, Page 2)

Thread the string about 1 inch through the hole in the shaft supporting the rotor. Turn the rotor so the string winds around the shaft – then you will pull on this to make the rotor spin. With the rotor spinning in the vertical plane (not as shown), gently push down on one side of the gimbal. What happens? Why?



Try this for two other orientations (change spin axis and direction of the moment). Let everyone in the group try this (you may have to spin it up again). Describe what you did and what happened. Feel free to sketch a figure with vectors if it helps. Describe in words what is happening with regards to the angular momentum.

Spin the rotor up again. Let the rotor point at some oblique angle (not vertical or horizontal). Hold the base and slowly move it around – change its orientation. The orientation of the rotor should point the same direction even if the base moves since there is no moment applied to the rotor (due to the gimbals). This works particularly well with the motorized one we had in class since the friction is so low in the bearings. This is generally how gyroscopic navigational devices work. List a few applications where this might be useful.

Toy Gyroscopes (Toy on the schedule, Page 2)

Thread the string about 1 inch through the small hole in the rotor shaft. Spin the rotor so the string winds around the shaft, then pull the string to get the rotor spinning. Have each team member hold the bottom of the gyroscope at Point A, then move the gyroscope around. You should be able to feel the gyroscopic moment as you turn it around.

Spin the gyroscope again, then place it on the small plastic stand provided. What way does it precess? Draw a sketch, explain what is happening using the gyroscopic formula. What happens to the precession as the spin of the rotor slows down?

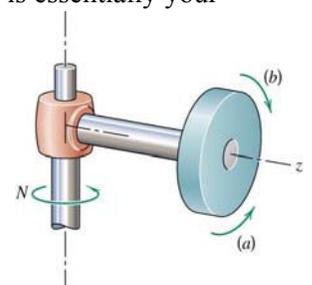


Gyroscopic Bicycle Wheel (Bike on the schedule, Page 2)

Have one person on the team hold the wheel as shown, and a second person push down on the wheel to get it spinning. Pirouette around like Baryshnikov- what do you feel?



Now hold one of the handles with the other handle pointing straight away from you. You probably need to get it spinning again. Precess around again – what direction does the wheel make your arm move? Again, show how and why this happens using a sketch and appropriate equations. This is essentially your online quiz problem.



Finally, share one of the rotating platforms with the other bicycle wheel groups. You can either stand on the large one or sit on the one on the stool. You will need to get the wheel spinning fairly fast and do the demo right after it starts spinning. Hold the wheel in front of you like shown in the picture. Tilt the bike wheel to the left about 30 degrees (if you did this through a full 90 degrees it would be horizontal). What happens? Move it back. Describe what and why this happens. If the rotor has constant spin speed, how might a spinning rotor be used to help orient a satellite?



With the different groups in the bike one, spin the wheel with the string attached. Then hold onto the string. What happens and why?