AC 2008-393: RIGID BODY DYNAMICS IN THE MECHANICAL ENGINEERING LABORATORY

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Rigid Body Dynamics in the Mechanical Engineering Laboratory

Abstract

This paper describes a relatively simple method in which planar rigid body motion can be measured and analyzed in the context of an upper division mechanical engineering laboratory course. The overall intention of this work is to help facilitate upper division level laboratory projects in dynamics. Such projects are intended to provide students with the opportunity to i) apply and reinforce their knowledge of dynamics, ii) learn and practice modern experimental methods used to make and assess motion measurements, and iii) if possible, compare theoretical and measured results.

The instrumentation involves the use of two inexpensive sensors – a dual axis accelerometer and a rate gyro – and a data acquisition system (such as LABVIEW). The accelerometer and rate gyro are fixed to the rigid body object. The rate gyro measures the planar angular velocity, which can be integrated with respect to time to yield angular orientation of the rigid body. With the use of this measured angular orientation, the accelerations measured by the accelerometer (which are measured in two directions *fixed to the rigid body*) can be resolved into directions *fixed in space*, and consequently integrated to yield velocity and position coordinates of the point on the rigid body where the accelerometer is attached.

These experiments also highlight the importance of error estimation. Errors in acceleration and angular velocity measurement lead to errors in angular orientation and translational velocity and position components that generally grow with time (due to the time integration). An estimation of acceleration and angular velocity measurement errors can be made by calibrating the accelerometer and rate gyro using simple homemade devices.

After describing how these experiments can be set up and performed in general, this paper will describe a specific experiment done in the author's junior mechanical engineering laboratory course. The rigid body object under study is a remote controlled car. The technique described above is used to find the position, velocity, orientation, and angular velocity of the car as a function of time. Results of the measurements and data analysis are compared with observations of the car's motion viewed by a video camera. An examination of this laboratory experience, with a discussion of intended learning objectives, an assessment of whether they are being achieved (based on surveys), and suggestions for improvement, will be included.

The paper will conclude with some suggestions for additional rigid body motion experiments using this general method of motion measurement.

I. Introduction

Engineering educators have identified several learning objectives to be achieved in engineering laboratory courses, including (but not limited to) the development of the following abilities (quoted from 1):

1. (Instrumentation) The ability to apply appropriate sensors, instrumentation, and/or software tools to make measurements of physical quantities.

2. (Models) The ability to identify the strengths and limitations of theoretical models as predictors of real world behaviors...

3. (Experiment) The ability to devise an experimental approach, specify appropriate equipment and procedures, implement these procedures, and interpret the resulting data to characterize an engineering material, component, or system.

4. (Data Analysis) The ability to collect, analyze, and interpret data, and to form and support conclusions...

5. (Communication) The ability to communicate effectively about laboratory work with a specific audience, both orally and in writing....

6. (Teamwork) The ability to work effectively in teams....

An additional objective of mine is to reinforce and deepen students' knowledge of core subject theories in the mechanical engineering (ME) curriculum.

Dynamics is not a traditional laboratory subject in the ME curriculum, probably because the analysis of problems follows deductively from a pair of hypotheses (laws of linear and angular momentum) which are time honored and well accepted. However, in recent years, there have been several efforts to introduce it into the laboratory. Most of these efforts involve the measurement of one dimensional particle motion (using an accelerometer or LVDT) or planar rotation about a fixed point (using an optical encoder or RVDT).ⁱ (See, for example, ^{2,3,4}.)

The attempt of this paper is to facilitate experimentation with planar rigid body motion (translation and rotation) in the ME laboratory. Such experiments can meet the objectives above.

The instruments used are a dual axis accelerometer and a rate gyro, which are cheap and easy to implement if it is assumed that one's program is already using basic data acquisition hardware and software. Furthermore, the instrument package can be assembled in a modular way so that the instruments can be easily transferred to different rigid body objects for measurement.

Section II of this paper briefly describes some of the relevant theory underlying accelerometer and rate gyro measurement and data analysis. Section III describes the basic instrument module, the data acquisition system, and the calibration procedures, independently from consideration of the rigid body object under study. Section IV describes a laboratory experiment performed in the author's junior mechanical engineering laboratory course, where the instrument module is mounted to a remote controlled car to measure its motion. An assessment of the experiment is also presented. Section V suggests other applications and Section VI offers a short conclusion.

ⁱ LVDT and RVDT stand for Linear and Rotary Variable Differential Transformer.

II Accelerometers and Rate Gyros

<u>Accelerometersⁱⁱ</u>

An accelerometer is designed to measure acceleration. Actually, it measures *acceleration relative to specific body force* (body force per unit mass). In particular,

$$a_n - b_n = K_a V, \qquad (1)$$

where a_n is the acceleration in a direction n, b_n is the specific body force in the n direction, V is the voltage sensed, and K_a is a calibration constant.

Typically, the body force is gravity and b_n , the body force per unit mass, is the component of g (gravitational acceleration) in the n direction. For example, an accelerometer pointed downward on a freefalling non-rotating object would measure 0 output.

If the component of gravitational acceleration b_n remains constant, then it can be zeroed out with an offset. Furthermore, if the accelerometer is measuring acceleration in the horizontal plane, then $b_n=0$.

Rate Gyros

A rate gyro measures angular velocity ω about a certain direction.ⁱⁱⁱ In particular,

$$\omega = K_{\omega} V \tag{2}$$

where V is the voltage sensed and K_{ω} is a calibration constant.

Planar Motion

Two perpendicular accelerometers (or a single dual axis accelerometer) and a rate gyro can be used to measure planar motion of a rigid body in the xy plane, as shown in Figure 1. The perpendicular accelerometers (or a dual axis accelerometer) are fixed to point P on the rigid body with the sensing directions 1 and 2 fixed to the rigid body and in the xy plane. Thus the accelerometers measure the body-fixed acceleration components a_1 and a_2 of point P. The rate gyro can be attached anywhere to the rigid body, although for practical purposes it makes sense to package it with the accelerometers. The rate gyro is aligned to measure angular velocity ω about the z direction (pointing out of the page).

ⁱⁱ Background on accelerometers can be found in ⁵.

ⁱⁱⁱ A rate gyro works by measuring Coriolis acceleration. For more on rate gyros, see ⁶.



Figure 1: Planar Motion Kinematics

The rigid body's angular velocity ω measured by the rate gyro can be integrated with respect to time to obtain the rigid body's angular orientation θ at time t:

$$\theta = \theta_0 + \int_0^t \omega dt \tag{3}$$

Here, θ_0 is the initial orientation at time t = 0. Once angular orientation at time t is determined, the *space-fixed* accelerations a_x and a_y of point P can be determined:

$$a_{x} = a_{1}\cos\theta - a_{2}\sin\theta$$

$$a_{y} = a_{1}\sin\theta + a_{2}\cos\theta$$
(4)

These can be integrated to yield *space-fixed* velocities of point P at time t:

$$v_{x} = v_{x0} + \int_{0}^{t} a_{x} dt$$

$$v_{y} = v_{y0} + \int_{0}^{t} a_{y} dt$$
(5)

which, in turn, can be integrated to yield the x and y position coordinates of point P at time t:

$$x = x_0 + \int_0^t v_x dt$$

$$y = y_0 + \int_0^t v_y dt$$
(6)

Again, the subscript 0 indicates initial components of velocity and position at time = 0.

III Experimental Setup

Instrument Module

The accelerometers and rate gyro can be assembled into the module shown in Figure 2.



Figure 2: Instrument Module

This module includes a dual axis accelerometer (ADXL213) and a rate gyro (ADXRS401) made by Analog Devices^{iv}, a 9 Volt power source with a 5 Volt regulator, an on/off switch with an indicator light, and two capacitors (One for filtering, one for timing). The outputs of each of the three devices are differential analog voltages which are transmitted via a long cable to a data acquisition system.^v The accelerometer and rate gyro are attached to evaluation boards which plug into the PC board. Thus they can be easily removed and replaced with other instruments

^{iv} These particular instruments are suitable for the RC car application discussed in Section IV, which requires the capability to accurately measure small accelerations (< 1 g) at low frequencies (down to DC).

^v It is possible to build a wireless system as well. Contact the author's technician (via the author) for more information.

with different sensing properties. The alignment of the accelerometer and the rate gyro in the module is critical.

The components are assembled in a case made by Bud Industries. The entire module can be bolted (via the mounting holes) to the rigid body object of interest. (The horizontal bar shown in Figure 2 is for mounting on the remote controlled car described in Section IV and does not have to be part of the module in general.)

The total cost of the module shown in Figure 2 (including the instruments, the electronics, packaging, and cable) is approximately \$60. The accelerometers can be purchased preassembled with the evaluation board which is a little more expensive but makes the assembly of the module easier.

The data acquisition system is shown below in Figure 3. It consists of a computer on a portable cart. The computer has a National Instruments data acquisition card which is hard wired to a breakout box containing terminals for the three pairs of wires in the cable. LABVIEW is used to acquire, condition, and process the data.

Readers interested in more details on the hardware are encouraged to contact the author.



Figure 3: Data Acquisition System

Calibration

As specified in Equations (1) and (2), the two accelerations and the angular velocity are proportional to the voltages sensed by the instruments. The calibration constants can be determined using the following simple procedures.

The dual axis accelerometer is calibrated using "static calibration". For each of the two axes, the accelerometer is held static with the axis inclined at various known angles with the vertical. Recalling Equation (1), $a_n = 0$ while b_n is specified at different values between -g and g.^{vi} For each inclination, voltage is obtained by the data acquisition system. Then the measurand $a_n - b_n$ can be plotted versus the voltage V and the calibration constant K_a can be determined from a straight line fit. Figure (4) shows a simple setup for this calibration, using a wooden plank resting on a crate for the incline, and a protractor to measure the inclination.



Figure 4: Calibration of the Accelerometer

For the calibration of the rate gyro, a turntable with adjustable speed would be an ideal setup, where ω could be specified and V measured. (See Equation (2).) However, a cheaper method is to specify the *average* angular velocity over a specified amount of time and measure the *average* voltage over the *same time period*. Average angular velocity is specified by turning the instrument module a specified angle over a specified time. (Average $\omega =$ total angle/total time.) This can be done by drawing lines at various angles on a piece of cardboard and using nails as stops to ensure that the total angle turned during the specified time is the specified amount. (See Figure 5.) The data acquisition system obtains an average voltage over the same time period. Values of ω are then plotted versus values of V, and the calibration constant K_{ω} in Equation (2) is obtained from a straight line fit.

^{vi} For the applications considered here, this calibration range is sufficient. For applications with much higher accelerations, a different calibration procedure would need to be employed.



Figure 5: Calibration of the Rate Gyro

Measurement Uncertainty

An important part of the mechanical engineering lab experience (see Objectives 3 and 4 in Section I) is the estimation of uncertainty in measured results.

The calibration procedures described above, in addition to providing values of the calibration constants, can be used to estimate the uncertainties of the measurements of the two accelerations a_1 and a_2 and the angular velocity ω . As the calculations of Equations (3)-(6) are performed on the measurements a_1 , a_2 , and ω , uncertainty is propagated through the calculations. These uncertainty calculations are quite extensive and are not presented here. Instrumentation texts such as ⁵ can be consulted regarding these procedures. It is important to note that when integration is performed, the uncertainties generally increase with time. Thus, the uncertainties in all of the results obtained from (3)-(6) increase with time. This establishes a limitation on the accuracy of this type of motion measurement but also provides an instructive lesson to be gained from it.

IV Application: A Remote Controlled Car

As an example application, consider the following experiment which has been conducted in the author's Material/Mechanical Laboratory course. This is a junior year capstone laboratory course in mechanical systems.

The goal of the experiment is to measure the motion of a remote controlled car with the instrument module described above. The instrument module is provided to the students. They

are required to calibrate the instruments, write the LABVIEW software, perform the data analysis, calculate uncertainties of all calculated results, and draw conclusions.

Figure 6 shows the remote controlled car with the instrument module attached. The car is made by DuraTrax and costs about \$120.



Figure 6: RC Car with Instrument Module

After calibrating the instruments and preparing the LABVIEW software, students run several tests of the car in a large room, including straight and curved paths with increasing and decreasing speed. Tape is laid down in the x and y directions with marks made every 1 meter. A video camera is set up to record the car's motion, along with a stopwatch started at the start of the data acquisition. (See Figure 7.) The videotape is played back in slow motion and the x and y position and orientation are estimated at several different times.^{vii} This provides a rough observation of the car's motion for comparison with the measured results.^{viii}

After the data is taken, students import the data into MATLAB to conduct the analysis described in Section II.

A set of sample results is shown below. Figure 8 displays the measured body-fixed accelerations $(a_1 \text{ is forward and } a_2 \text{ is left})$ and angular velocity. In this particular run, the car accelerates forward, then turns left and decelerates. (See observed positions and orientation in Figures 9₁, 11, and 12) Note that the data is quite noisy. Much of this is the mechanical vibration induced

^{vii} The videotaping procedure has not been implemented in the author's class yet. It was developed by the author since the last class, and will be implemented with the next class.

^{viii} More sophisticated video equipment, along with more grid lines, would provide a more accurate observation.

by the car's motor.^{ix} Note also that some data from the beginning of the test has been truncated. This is because the car hasn't started moving yet, and small offset errors lead to large errors after integration.



Figure 7: Snapshot of Video at the Start of a Test



Figure 8: Instrument Outputs: Body-Fixed Accelerations and Angular Velocity

^{ix} The data has actually been filtered numerically in LABVIEW with a low pass 2nd order Butterworth filter at 20 Hz.

Figure 9 displays the orientation θ and the space-fixed accelerations a_x and a_y after Equations (3) and (4) are applied. The initial orientation is taken to be 0. In addition, the observed orientation at various times is plotted. Note that the measured orientation tracks the observed orientation rather closely, although there appears to be a slight time lag.



Figure 9: Orientation and Space-Fixed Accelerations

Figure 10 displays the fixed axis velocity components v_x and v_y , (using zero initial velocities) as well as the speed $v = \sqrt{v_x^2 + v_y^2}$. Note how the integration of the noisy accelerations yields smooth velocities. The speed returns to zero, which was observed during the test.

Figure 11 displays the position coordinates, along with the observed position coordinates. Note that, like the orientation data mentioned previously, the observed x and y positions tend to 'lead' the measured positions. Some of this error may be due to a slight lag in starting the stopwatch for the video camera.

To help the students interpret their results, they are given a MATLAB function file which animates the motion of the car *according to their measured results*. For the data above, a snapshot of the animation, showing the path traveled and the orientation of the car, is displayed in Figure 12.^x Also shown are the observed x and y positions. The measurement of the car's path proved reasonably accurate. The animation (only a snapshot of which is shown in Figure 12) also shows that the orientation of the car follows the slope of its path (there is not much lateral slip), which agrees with observations from the videotape.

^x The interested reader is encouraged to contact the author for a copy of the function file or information about it. This work on animations follows the work of 7 .



Figure 11: Position Coordinates



Figure 12: Animation of Motion, Along With Observed Positions

As discussed in Section III, the estimation of uncertainty of the measured results is an important part of the laboratory experience. Here, the uncertainty of the angular velocity measurement is estimated (based on the calibration data) at .1 rad/s (at 95% confidence) and the uncertainty in the orientation $\theta(t)$ is determined. Figure 13 shows $\theta(t)$ zoomed in with + and - uncertainty bands drawn in. Note that even though the uncertainty remains small over the time shown, it is growing with time. The uncertainty in this case does not account for the error between measured and observed orientation. Other errors, such as alignment of the rate gyro, the time lag previously mentioned, and the approximation of the observed orientation from the videotape, are playing a more substantial role.

One of the challenges of implementing this experiment is the alignment of the accelerometers and rate gyro on the car. In addition, the car pitches and rolls slightly on its suspension as it accelerates and turns. This tends to *over-measure* the accelerations a_1 and a_2 . (The measured values tend to be too high) These errors can be significant, yet are not accounted for in the measurement uncertainty analysis described above. It may be possible to mitigate these errors by making the suspension more rigid or by employing more sensors to measure the full three dimensional rotation of the car. These errors did not seem to play a substantial role in the particular test presented above; however, it has shown up in other testing by the author and in some of the tests done by students.



Figure 13: Orientation with Uncertainty Bounds

Assessment of the Experiment

At the conclusion of the experiment in last year's course, students were issued a survey, which asked them to rate (from 1 to 5) the degree to which this experiment improved their abilities in four areas. The abilities and scores are tabulated below.

	This experiment improved my ability to:	Score (out of 5)
1	Install and calibrate accelerometers, estimate the	4.57
	uncertainty of their measurements, acquire acceleration	
	data electronically, and analyze and interpret data.	
2	Apply the theories of dynamics to a system under	4.19
	experimental study and assess the abilities of these theories	
	to predict results	
3	Present experimental procedures, theory, results, and	4.29
	conclusions clearly and efficiently.	
4	Work effectively in groups on conducting experiments,	4.48
	performing analysis, and preparing reports	

Clearly, according to the opinion of the students in this class, this experiment has advanced (to various degrees) the Objectives 1, 2, 4, 5, and 6 stated in Section I. One shortcoming of this particular experiment is the lack of a theoretical model with which to compare results; therefore the advancement of Objective 2 is not as strong as it could be. (The data analysis is based on the kinematics of rigid body motion. However, the laws of linear and angular momentum are not directly tested.) Other experiments can be devised which have theoretical solutions with which to compare measured results. (See Section V.)

Objective 3, which has to do with experimental design, could be more effectively addressed by giving students less guidance; for example specifying a lab objective (measure the motion) and letting the students figure out how to do it. The more experience they have with accelerometers and rate gyros and experimentation in general, the more open-ended the project can be.

The reinforcement of dynamics was not directly assessed. Surely, however, students gained additional exposure to rigid body kinematics.

V Other Applications

Section IV describes one experiment using the hardware described in Section III. Other experiments can be devised. For example, consider a compound pendulum, such as a Charpy Impact Tester that resides in many Materials Laboratories. The instrument module can be bolted to the pendulum, with the dual accelerometers measuring normal and tangential accelerations (of the point of attachment) and the rate gyro measuring angular velocity. Because the motion is in the *vertical* plane, the body force terms b_n of Equation (1) for each of the two accelerometers depend on the orientation angle θ of the pendulum; hence it is necessary to obtain θ first (by integrating the rate gyro output) and then use θ to obtain the actual accelerations a_n from the accelerometer outputs $a_n - b_n$.^{xi} The advantage of doing this experiment is that a simple analytical solution for the pendulum can be obtained (based on the law of angular momentum, and estimations of mass properties based on geometric measurements and estimated density values) which can be directly compared to the measured results. This is advantageous for two reasons: i) it gives students the opportunity to perform kinetics analysis, and ii) it gives them confidence in the laws of dynamics. (i.e., Objective 2 from section I)

Another idea for an experiment involving a translating *and* rotating rigid body (the pendulum rotates about a fixed point) and with a theoretical solution is the classic example of a bar falling in the vertical plane with one end sliding down a wall and one end sliding along the floor.

^{xi}It is possible to obtain $\theta(t)$ using just the tangential acceleration. However, the calculations involved are complex, involving the solution of a differential equation where the forcing function is the data input (See Equation (1)). The author has done this (obtaining accurate results) with a previous class but felt that the complexity of the calculations made the use of accelerometers seem very challenging and confusing for students. The use of the rate gyro should make this much more straightforward; therefore the author is planning on doing this experiment with an upcoming class.

VI Conclusions

A relatively simple and inexpensive instrument module can be assembled which can be used to measure the motion of rigid body objects. This can be used to do rigid body dynamics experiments in mechanical engineering laboratory courses, achieving common ME laboratory objectives while giving students experience in theoretical and experimental dynamics.

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